Optimal programming model based on the Aerial Refueling Flight Plan

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Abstract: This paper is mainly about the optimal planning problem of the aerial refueling plan. Firstly, we take the aircraft rescue on the desert island as an example. In this case, we supposed that the total success rate of the whole process of aircraft rescue is 100%. In light of the above, we derived a linear relationship between the number of auxiliary machines and the maximum flight radius. Also, we have got an optimal solution to the problem under the premise of the maximum range of the rescue aircraft, the navigation distance, the host fuel load, and the auxiliary vehicle fuel load. Next, the paper makes a further discussion on the problem mentioned above. That is to say, in the process of refueling, the success rate of the rescue cannot reach the ideal conditions (100%) that people have given it. At this time, it should be considered the influence on multiple factors of the total success rate during aerial refueling. So, the main reason is to consider the three factors of failure, namely the possibility of failure in-air refueling and the possibility of failure of transportation and tanker. By analyzing the possibility of failure of these three aspects as well as deriving four related theorems, we further derive the functional relationship between the number of auxiliary machines and the success rate. From the theory mentioned above, it turns a complex problem into a variable plan. Finally, a simulation with MATLAB can achieve a success rate of 99.6% or more and get the best results. From the experiment, it proves that the scheme has high optimization efficiency and the obtained the fueling scheme is also reasonable.

Keywords: diversified planning, maximum flight radius, minimum amount of the auxiliary aircraft

1 Preliminary modeling

1.1 Definition before model construction

In order to enable rescuers to complete the rescue mission, we first establish a viable air refueling plan. Before building the model, we will first define some concepts. The maximum flight radius r of the aircraft receiving oil refers to the farthest distance that the aircraft receiving oil can fly (and safely return) to base A with the help of n auxiliary aircraft. As shown in Figure 1;

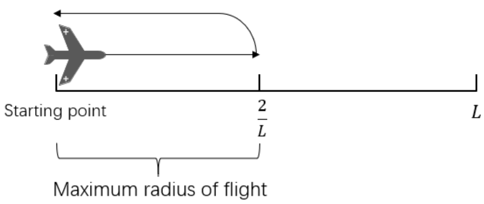

Figure 1. Aircraft Maximum Radius Definition Map

1.2 model construction

The following is the actual process of establishing the model:

First, we will make some assumptions (when we take the number n of different refueling machines, we will find out the corresponding values of the flight radius of the main engine) and then summarize the general rules. The analysis steps are as follows:

When $n=0$:

$$r=\frac{L}{2} \tag{1}$$

When $n=1$:

$$\begin{cases} \text{When k=1: } r=\frac{2}{3}L \\ \text{When k=0: } \begin{cases} r-S_1+r=L \\ 2S_1+S_1=L \end{cases} \\ S_1=\frac{1}{3}L,\ r=\frac{2}{3}L \end{cases} \tag{2}$$

When $n=2$:

$$\begin{cases} \text{When k=1: } \begin{cases} 2S_1+S_1=L \\ (r-S_1)+(r-S_2)=L \\ 2S_2+S_2=L \end{cases} \\ S_1=\frac{L}{3}, r=\frac{5}{6}L, S_2=\frac{L}{3} \\ \text{When k=0: } \begin{cases} r-S_2+r=L \\ 2(S_2-S_1)+S_2=L \\ 3S_1+S_1=L \end{cases} \end{cases} \tag{3}$$

$$r=\frac{3}{4}L, S_2=\frac{1}{2}L, S_1=\frac{1}{4}L$$

Based on the above solution, we deduce the general situation: the situation that k tankers are responsible for sending and receiving oil machines.

When $n=m$, k tankers are responsible for escorting planes receiving gasoline:

$$\begin{cases}(k+1)S_1+S_1=L\\ 2(S_k-S_{k+1})+S_k=L\\ r-S_k+r-S_n=L\\ 2(S_n-S_{n-1})+S_n=L\\ \vdots\\ (n-k+2)(S_{k+2}-S_{k+1})+S_k=L\\ (n-k+1)S_{k+1}+S_{k+1}=L\end{cases} \tag{4}$$

$$S=\frac{L}{k+2}, S_k=\frac{kL}{k+2}, S_{k+1}=\frac{L}{n-k+2}, \quad S_n=\frac{(n-k)L}{n-k+2} \tag{5}$$

$$r=(\frac{(n-k)L}{n-k+2}+\frac{kL}{k+2}+L)/2 \tag{6}$$

After simplifying the formula of r :

$$r=\frac{3}{2}L-\frac{L}{n-k+2}-\frac{L}{k+2} \tag{7}$$

To discuss its extremes, we derive the derivative number and derive the derivative result as:

$$r'=\frac{L(n+4)(n-2k)}{(k+2)^2(n-k+2)^2} \tag{8}$$

From its derivative form, we can find some beautiful things when the number of oil receiving aircraft is twice less than the maximum flight radius of the oil receiving aircraft when the oil delivering aircraft is operating, and conversely, when the number of oil receiving aircraft is twice more significant than the maximum flight radius of the oil receiving aircraft when the oil delivering aircraft is operating, the extreme value (as shown in Figure 2) is obtained here, that is, the maximum flight radius of the oil receiving aircraft.

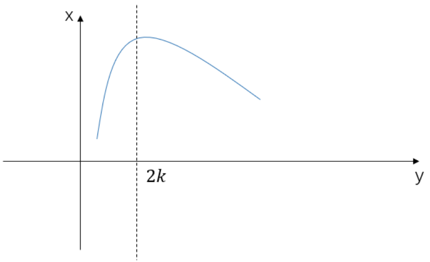

Figure 2. R's function image

1.3 Model discussion

When we consider the actual situation, k (number of fuel receivers) must be a positive integer.

Then we rewrite the previous formula on the maximum flight radius of the receiving aircraft:

$$r=\frac{3}{2}L-\frac{L}{n-k+2}-\frac{L}{k+2} \tag{9}$$

$$k=\frac{n}{2} \quad \text{When n is an even number} \tag{10}$$

$$k=\max\left\{(r_{\frac{n-1}{2}}),(r_{\frac{n+1}{2}})\right\} \text{When n is an odd number} \tag{11}$$

And then the corresponding restrictions establish the model:

$$\begin{cases}r\ge L\\ \dfrac{l\text{-}\dfrac{kL}{k+2}}{L}a \qquad \ge 50\text{-}15\end{cases} \tag{12}$$

(2) The fuel consumption from the last delivery point to the maximum flight radius is higher than the weight of the person.

After our model verification, we obtained the optimal navigation mode is shown below:

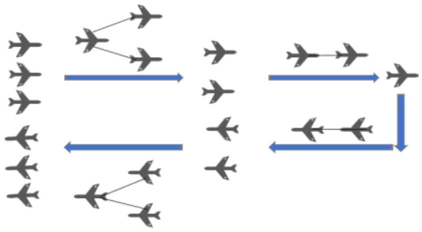

Figure 3. The optimal navigation mode

2 Optimization model

2.1 The optimization idea of the model

Based on the first question, we will make a further discussion on the problem above. Because aerial refueling is a high-risk operation and maybe there is a possibility of failure. Especially in transportation and refueling aircraft, we will then consider the probability of the failure each item to ensure that the total success rate has been increased as much as possible in order that the corresponding optimal solution is found.

2.2 The Proposition and Proof of Theoretical Model

The specific steps for building our model are as follows:

Theory 1: Whether the formula of n is an odd number or even number, when its maximum flight radius r is taken from the place of k , the maximum flight radius r was taken from the place of $n+1$ is from the functional relation

$$\text{when}\begin{cases}k\\ k+1\end{cases} \tag{13}$$

Lemma: When n is an odd number, the maximum flight radius r has the same result whether it is under the condition of $k=\frac{n+1}{2}$ or $k=\frac{n-1}{2}$.

Prove:

$$r=\frac{3}{2}L-\frac{L}{n-k+2} \quad (14)$$

When $k=\frac{n+1}{2}$:

$$r=\frac{3}{2}L-\frac{L}{\frac{n-1}{2}+2}-\frac{L}{\frac{n+1}{2}+2} \quad (15)$$

When $k_1=\frac{n-1}{2}$:

$$r_1=\frac{3}{2}L-\frac{L}{\frac{n-1}{2}+2}-\frac{L}{\frac{n+1}{2}+2} \quad (16)$$

Therefore, it has to be proved.

In the same way, we can also prove that the maximum flight radius can be calculated at the place of k.

Theory 1 mainly gets the conclusion that adding an aircraft can only change the times to refuel for the tanker, namely, add an aircraft when $k\le\frac{n}{2}$ and it can also change the times to refuel for the main engine by the refueling machine, namely, add an aircraft when

$$k\ge\frac{n}{2} \quad (17)$$

Theory 2: The probability of failure will increase if the refueling position point has been added.

Proof: Suppose a refueling point S_j is added, The starting probability before the refueling point is not increased is,

$$\partial^k\partial^{k-1}\cdots\partial\partial\partial^2\cdots 1-\partial^{n-k}$$

After adding refueling points, we can see from Theorem 1 that adding an aircraft will add a point so that the total number of refueling does not change.

$$\partial^{k+1}\partial^k\partial^{k-1}\cdots\partial\partial\partial^2\cdots\partial^{n-k} \quad (\text{when } k\le\frac{n}{2}) \quad (18)$$

Similarly, the probability $k\ge\frac{n}{2}$ can be obtained. The probability of failure increases, so Theorem 2 is correct.

Therefore, on the premise of minimum refueling points, the success rate will be the highest. According to the first question, the refueling point is 4. At this point, the original problem is reduced to seeking the maximum success rate on the premise of minimizing the number of refueling points.

Theory 3: Adding a tanker to refuel the first aircraft at the first refueling point will increase the success rate.

Proof: The success rate when the aircraft is not increased is increased, and the success rate after the increase is:

$$p_1=(1-(1-\partial)^{k+1})) \quad (19)$$

Therefore, theory 3 is correct.

Theory 4: If the number of second refueling is k, the number of first refueling aircraft is $\frac{k(k+1)}{2}$.

Proof: because the number of second tankers is k, the last time is $k+1$, the number of the first aircraft is $\frac{k(k+1)}{2}$, and get the certificate.

2.3 Discussion on model

From theorems 1, 2, 3, and 4, the following inferences can be drawn:

Increase the number of tankers for each receiver at each refueling point, assuming that each receiver has k tankers to refuel it, the success rate is

$$p_{\text{sum}}=(1-(1-\partial)^k)(1-(1-\partial)^k)^k(1-(1-\partial)^k)(1-(1-\partial)^k)^k \quad (19)$$

Therefore, the final model is

$$\begin{cases} p_{\text{sum}}=(1-(1-\partial)^k)(1-(1-\partial)^k)^k(1-(1-\partial)^k)(1-(1-\partial)^k)^k \\ n=\frac{k(k+1)}{2}+k \\ p_{\text{sum}}\ge 99\% \end{cases} \quad (20)$$

The approximate operation route of this optimal situation is shown in pic.2.1:

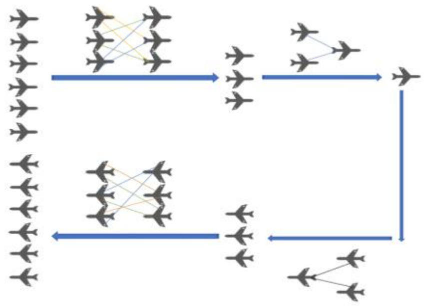

Figure 4. The approximate operation route of this optimal situation

3 Mode Solution

By modeling with MATLAB, we take different values of n and find out that the minimum n that satisfies the condition is the result we want to get, by modeling, we get $n=4$ as the minimum n satisfies the condition. At this time, the corresponding value k indicates the result of 4.

The following is the interface for building the model:

When n is an odd number:

We gave it a qualitative description through Excel in the attached table as follows:

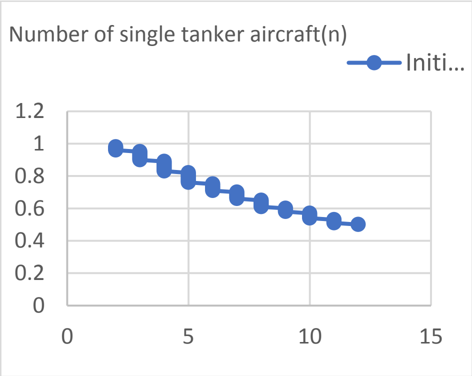

Figure 5 (a) The relationship between the number of single tankers (n) and the initial success rate (m)

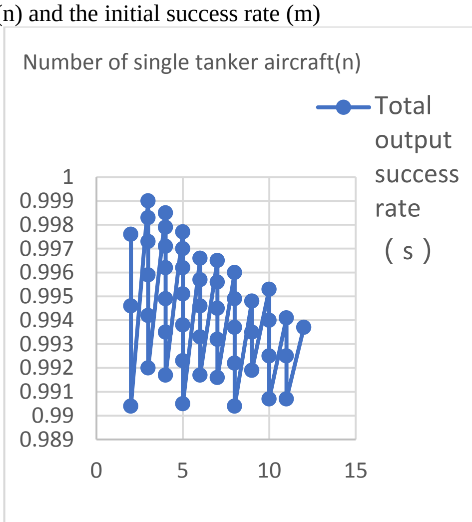

Figure 5 (b) The relationship between Number of single tanker aircraft(n) and Total output success rate(s)

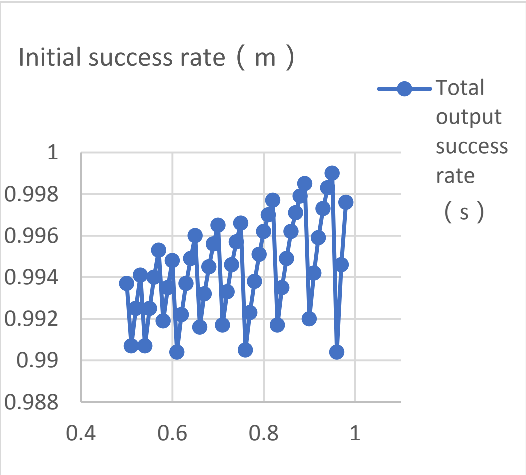

Figure 5 (c) The relationship between Initial success rate (m) and Total output success rate(s)

4 Conclusions

This model establishes a high-efficiency solution of aerial refueling flight plan, its advantages lie in that it can solve this tricky worldwide problem by using the pure mathematics calculation. What is more, it can establish a pluralistic security analysis mode. After reading relevant documents in the past, we have found out that the paper in the past put most emphasis on how to establish high-efficiency aerial refueling flight plan, but they do not make an intensive study of security analysis mode which is also the strong point in our essay. For a reason mentioned above, we analyze different kinds of issues about the influence on the success rate of aerial refueling flight plan by using pluralistic considering, which is also the topic of the essay. This has never existed before. By using our optimized overall operation mode and taking the success rate of every link into consideration so that we can get a complete operation mode which has the success rate more than 99%,but there are also some shortages during this process. That is when we analyze the total success rate, the sufficient data are restricted, therefore, it's, not all-round when considering the valid data. That is also an emphasis which we will study next. We hope we can have profound breakthroughs in this aspect shortly.

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